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THE HEIGHT OF THE LEVEL OF MAXIMUM WIND
AS A FUNCTION OF THE HEIGHTS OF THE
TROPOPAUSE AND THE 300-MB SURFACE

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as a Function of the Heights of the
Tropopause and the 300-mb Surface

by

Donald N. Brown

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
METEOROLOGY

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

A need for more knowledge of meteorological parameters in the region 30,000 to 45,000 feet has come about in recent years as a result of rapidly growing jet aircraft operations. This paper discusses one of these parameters, the height of the level of maximum winds, which affords considerable aid to the flight planner in his selection of preferred flight altitudes and routes. The deviations of the level of maximum winds from the tropopause are discussed and graphically shown, indicating that the latter is not a satisfactory first approximation to height of the former. This fact gives rise to the development of a series of simple and multiple regression equations for determining the height of the level of maximum winds as a function of both the tropopause and the 300-millibar heights.

Professor Robert J. Renard of the U. S. Naval Post-graduate School has been the guiding force behind this investigation. For his time and effort, the writer is deeply indebted.

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1. Introduction.

With the advent of jet aircraft in recent years, the forecaster's spatial envelope of responsibility has almost doubled. It has become necessary to analyze the meteorological parameters up to 100 mb on a daily basis. Knowledge of wind, cloud, temperature, and pressure fields at these levels are necessitated by the demands of aircraft flight planning. The United States Weather Bureau [9] has suggested that the forecast accuracy of the temperature and wind at jet flight levels be $\pm 3^{\circ}\text{C}$ and ± 20 knots respectively.

Not the least important of the parameters needed for economical operation of aircraft over long distances are the wind speed and direction, including their variation both in time and space. Over the continental United States and parts of Canada, a United States Weather Bureau analysis of the layer of maximum wind (hereafter abbreviated LMW and used synonymously with level of maximum wind) is available on a 1/40,000,000 polar stereographic facsimile chart every 12 hours. The height of the LMW typically varies from about 24,000 feet in the northern sections to about 44,000 feet in the south. This analysis, along with the contour analyses of the standard isobaric surfaces, reasonably describes the horizontal wind field and its derivatives throughout the high troposphere and lower synoptic stratosphere.* However, the extension of the LMW analysis, particularly

* a term defined by H. A. Panofsky [10], which includes the region from the tropopause to about 10 millibars.

to ocean areas, can be accomplished only with great difficulty owing to the paucity of wind data in the stratosphere. Thus, some means of determining with reasonable accuracy, the LMW data from the more easily obtained variables at lower levels would allow extension of the high-level wind analysis to ocean areas. One phase of the problem involves determination of the height of the LMW. It is this parameter which is under statistical investigation here.

2. Background.

Much has been written concerning the environment of the jet aircraft since the inauguration of high altitude, long range flights within the past ten years. Relative to the specific problem at hand, Reiter [5] discusses many aspects of the layer of maximum winds, including reliability of upper-wind data, analysis of the horizontal and vertical structure of the LMW, and forecasting the LMW, with special application to jet flight operations. His general conclusions testify to the need for not only more accuracy in upper-wind measuring devices, but to their proper representation and interpretation as well.

Since there are rather large inaccuracies in the upper-wind data and because of the scarcity of data in certain areas, it becomes desirable to determine the LMW from more commonly available data. The United States Weather Bureau assumes that, for purposes of analysis, the height of the LMW is at, or very nearly at, the tropopause in middle and northern latitudes. The tropopause, if considered a first-order discontinuity in the temperature field, is logically where one would expect to find the maximum wind. Unfortunately, as will be shown later, there are many occasions in which the LMW deviates significantly from the tropopause height. Johannessen [2] discusses a series of observations taken at Larkhill, England, which indicates that the LMW occurs from 2000 to 4000 feet below the tropopause. However, extension to other latitudes is not plausible.

The investigative results whose discussion follows is a

consequence of the actual deviations of the height of the LMW from that of the tropopause over the United States during the months of November 1959 and January 1960, which include a great variety of weather types.

3. Deviations of the Height of the LMW from the Height of the Tropopause

The tropopause definition as used by the United States Weather Bureau [11] is, using only the lapse-rate criteria, that point in the sounding at which the lapse rate decreases to $2^{\circ}\text{C}/\text{km}$ and then averages less than this in the 2-km layer immediately above.

Whether nationally or internationally defined, the tropopause level is a controversial subject. Stinson [8] in recently proposing a new tropopause identification and classification system deals with a myriad of soundings which bring out the difficulties in selecting a certain point fulfilling the above lapse-rate condition.

Regardless of inaccuracies or uncertainties of tropopause location, the heights as given on the United States Weather Bureau 00Z and 12Z facsimile charts of tropopause heights (Z_T) were used. These charts were analyzed by the author for use in this investigation. A typical example of the analysis is shown as part of Figure 1. A striking feature is the band of strong height gradient oriented generally east-west from southern California through Texas to Florida. This represents, for the purpose of this study, a continuous zone of transition between the tropical and middle-latitude tropopauses. Undulations of this band are always present in varying degrees of amplitude and are associated with the major weather systems. For example, the passage of a low or trough at 300 mb is synonymous with a trough in the tropopause heights. Another zone of large height gradient appears in northwest United States and is probably associated with the transition from the

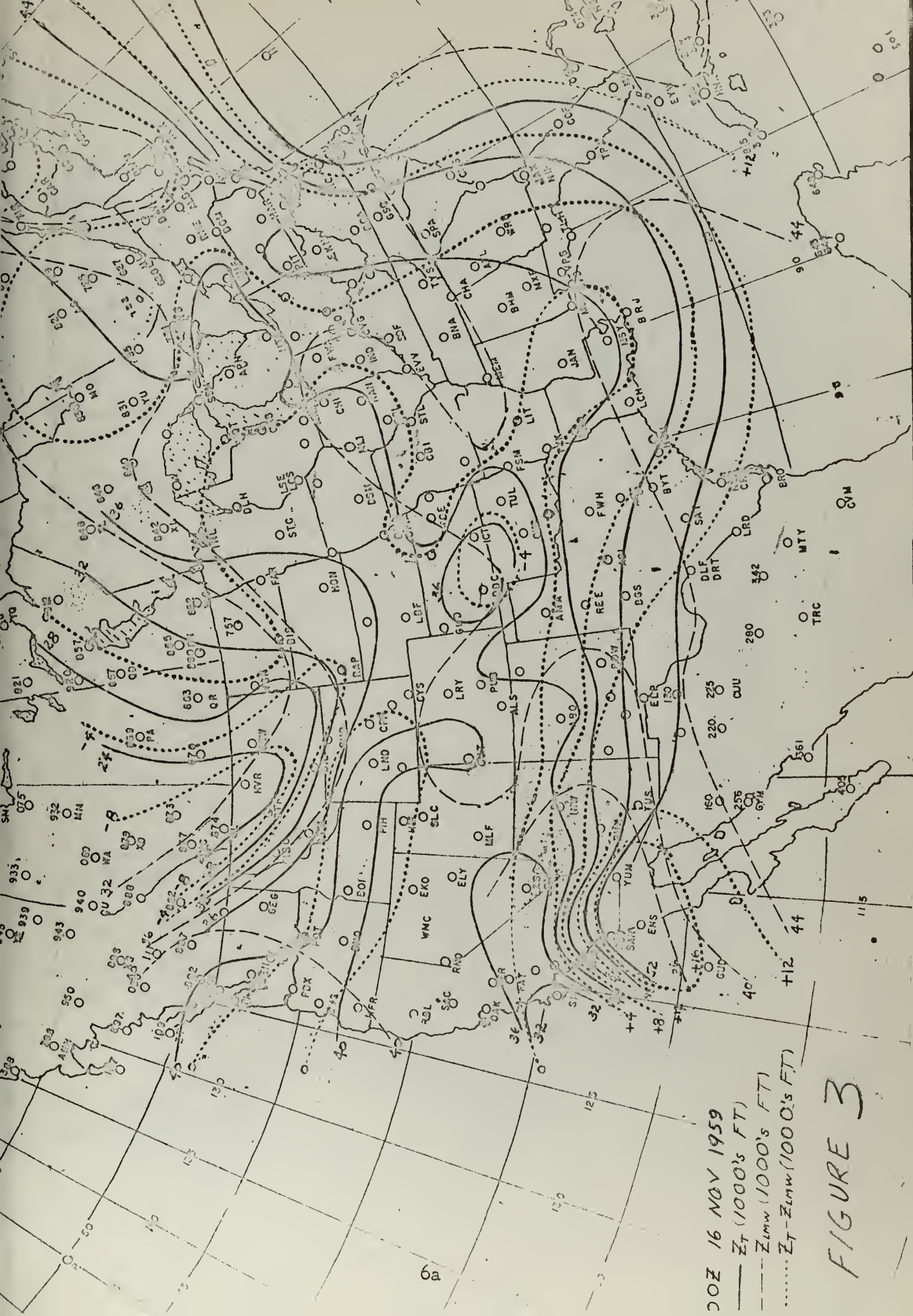
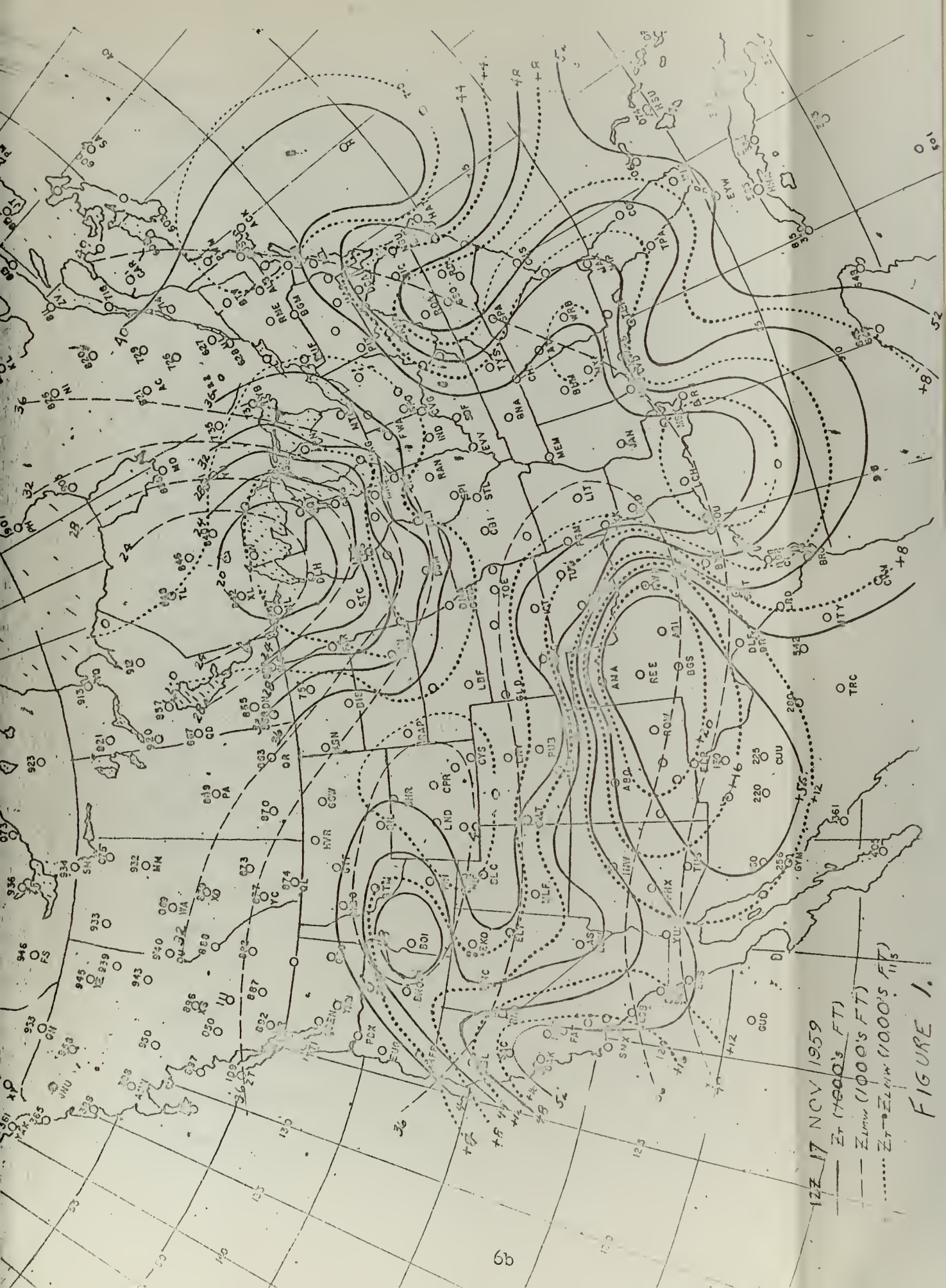


FIGURE 3



12Z 17 NOV 1959

— Zr (1000'S FT)

- - - Zm (1000'S FT)

..... Zr-Zm (1000'S FT)

FIGURE 1.

middle to the polar tropopause. A hemispheric analysis of this same phenomenon [3] shown in Figure 2, clearly exhibits the nature of the two transition zones just mentioned.

As an approach to defining statistically some relations between the height of the LMW (Z_{LMW}) and the tropopause, the United States Weather Bureau analysis of the height of the LMW (see Figure 1) is compared to the locally produced tropopause analysis by a graphical subtraction of the contour heights of the two surfaces. This also is shown in Figure 1. The general area of approximate coincidence of the two surfaces extends from extreme northwest United States toward the southeast through Missouri and from extreme northeast United States toward the southwest through Tennessee. In general, this area of coincidence, defined as that section in which $Z_T - Z_{LMW} \approx 0$, follows closely the 36,000 to 40,000-ft channel in the tropopause field. In areas of tropopause height less than 36,000 feet, $Z_{LMW} > Z_T$ while tropopause heights greater than 40,000 feet are associated with $Z_{LMW} < Z_T$. Further, since the tropopause is closely associated with major weather systems, it is reasonable to suspect that the troughs, ridges, and centers in Figure 1 will have continuity from day to day. That this is true is shown in Figure 3, which is a graphical subtraction for the situation as it existed 36 hours previous to that in Figure 1. The movement of the minus eight isoline over Montana corresponds closely to the movement of a 27,600-ft low at the 300-mb surface situated about 300 miles

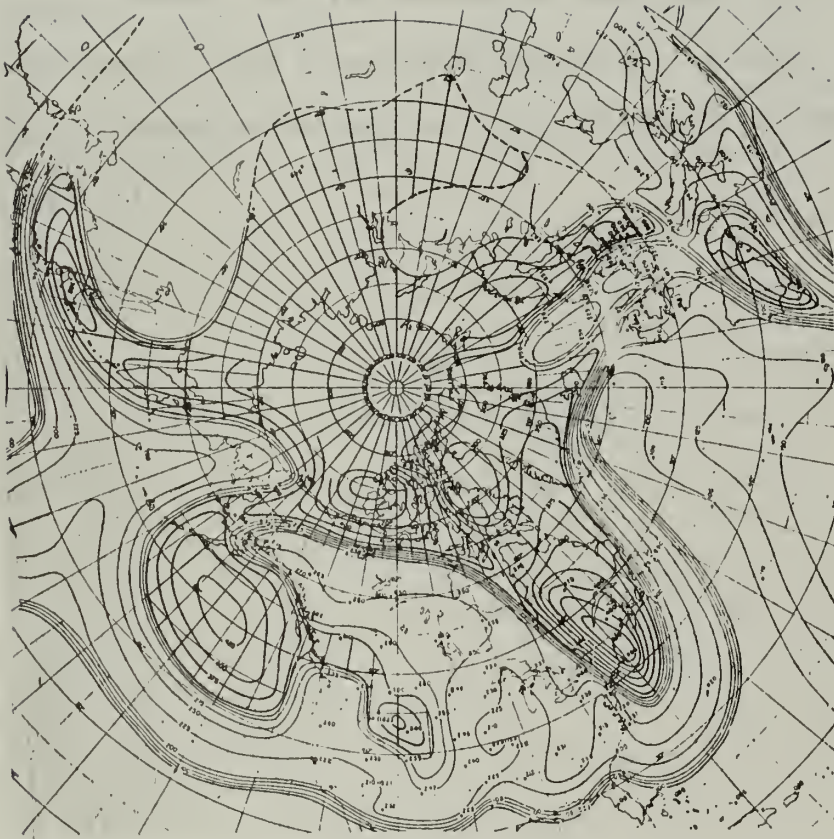


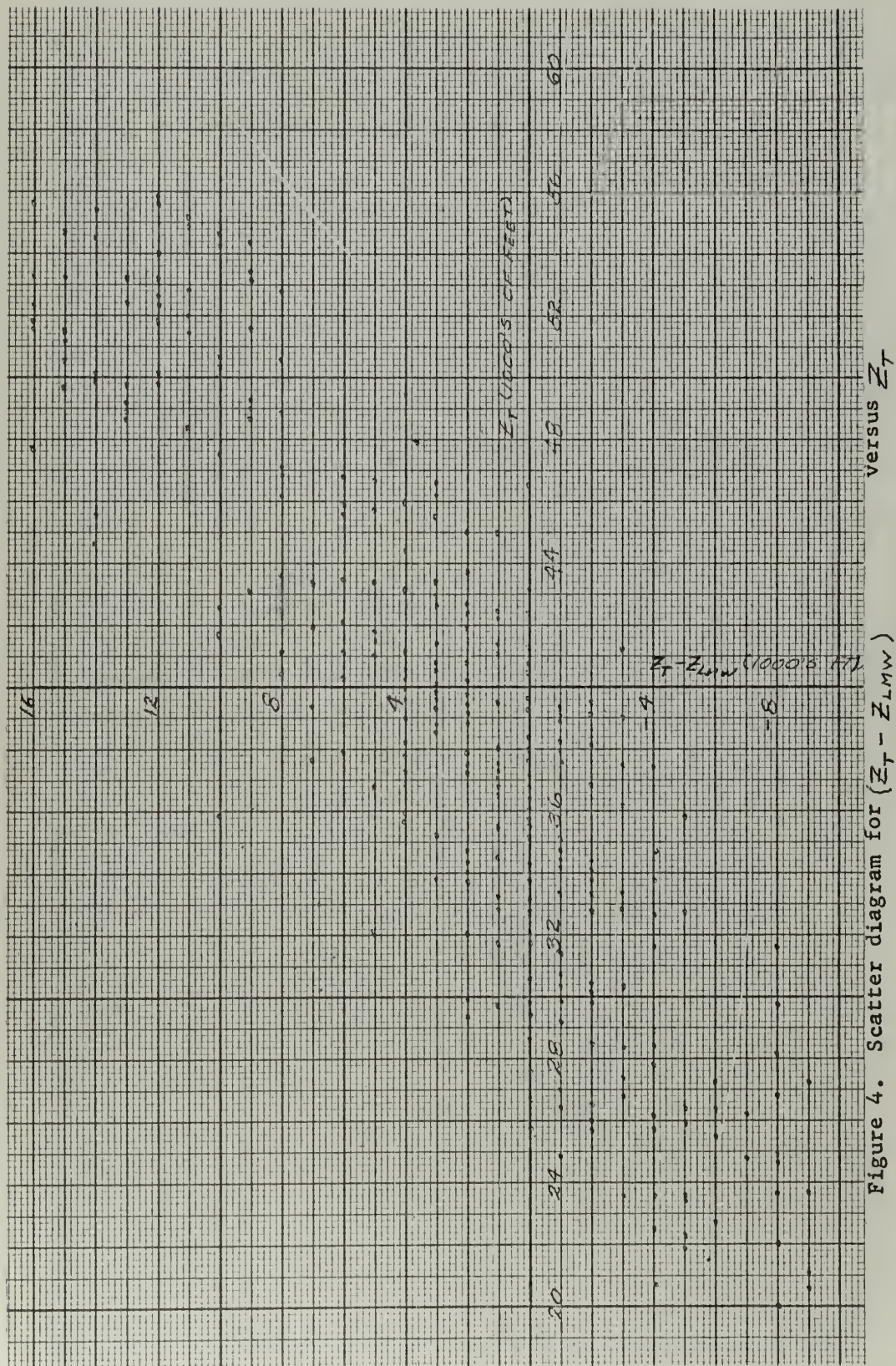
Fig. 2 Tropopause Height Contours Jan. 1, 1956.
(After Defant and Taba.)

to the north. The plus 16 area moved east and increased to a plus 20 over Texas with another plus 16 coming in over central California on Nov 17. These large plus areas appear to follow the movement of a 300-mb ridge located 400 miles to the west. Thus, figures 1 and 3 show that an assumption of Z_T equalling Z_{LMW} can be made only over a limited area of the continental United States. For the two months considered, the tropopause was found to vary from 12,000 feet below the LMW in northern United States to as high as 24,000 feet above the LMW in the south.

4. Statistical Relationship Between the Height of the LMW and other Parameters

A study of Figure 1 suggests a definite relationship between $(Z_T - Z_{LMW})$ and Z_T . A better picture of this relationship is shown in Figure 4 as a scatter diagram of Z_T versus $(Z_T - Z_{LMW})$, the points being randomly selected from November 1959 and January 1960. A best-fit line may be drawn through these points using the least squares method but because of the decrease in slope for the envelope of data below $Z_T = 36,000$ feet, the standard error would undoubtedly be excessive. Thus, two straight lines may be drawn, one for Z_T greater than 36,000 feet and the other for Z_T less than 36,000 feet. The discontinuity which occurs between the two regression lines at $Z_T = 36,000$ feet can easily be smoothed in to provide one continuous curve. Before proceeding on computational work, approximately 1000 actual data points, distributed quite evenly over the two-month period, were selected, and data for Z_T versus Z_{LMW} recorded. About half of these randomly-selected points were plotted on the scatter diagram of Figure 5. This diagram shows that a two-dimensional linear relationship will give best results for values of Z_T ranging from 30,000 feet to 44,000 feet. Above and below this range the slope of the data envelope changes and the scatter increases.

With a view toward simple utilization of all data points, it was felt that the best course of action was to work first with the data as originally divided, that is, for tropopause heights above and below 36,000 feet. In order to use the 1000 points



versus Z_T

Figure 4. Scatter diagram for $(Z_T - Z_{LMW})$

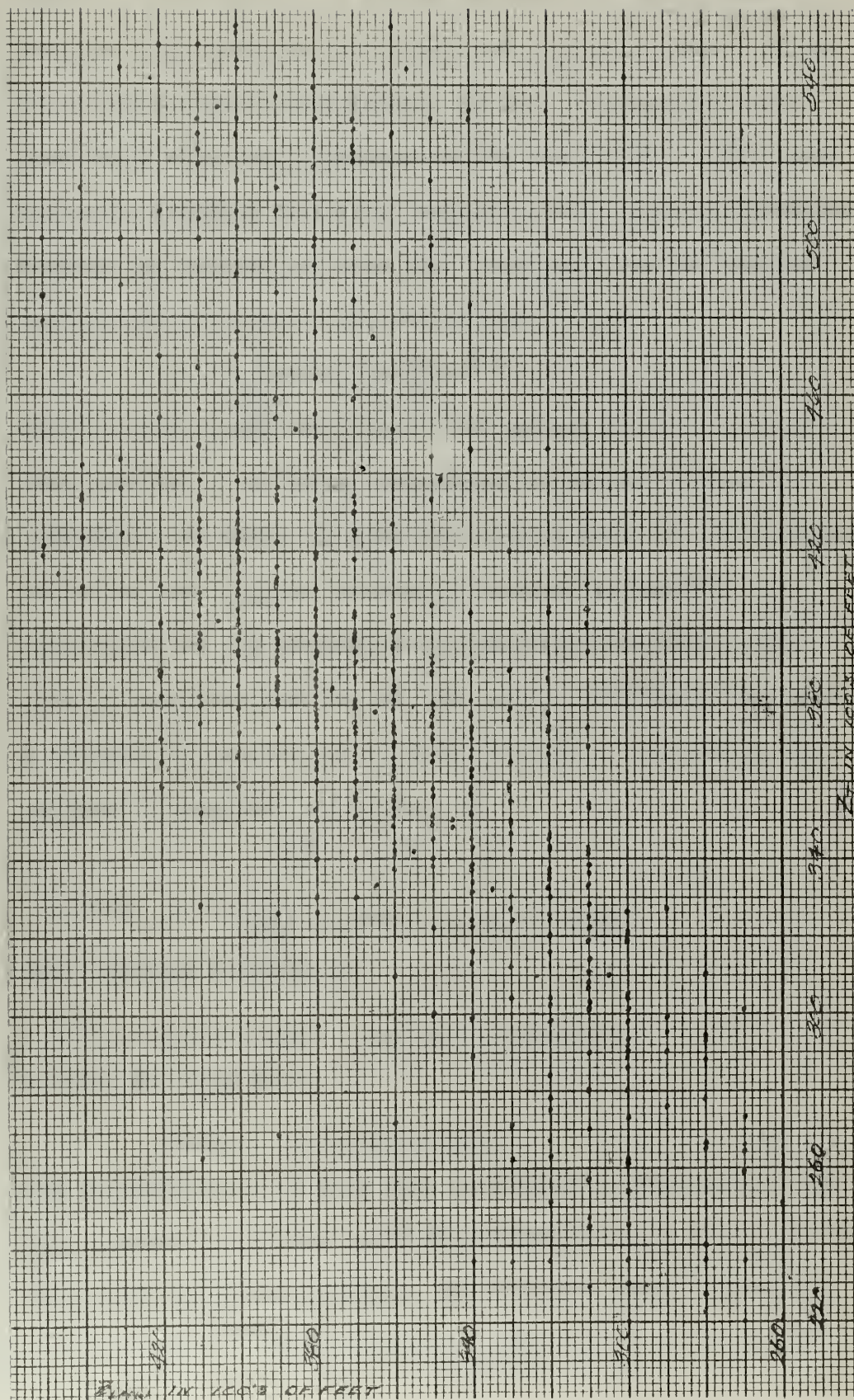


Figure 5. Scatter diagram for Z_T versus Z_{LMW}

taken over the period, it became necessary to employ the National Cash Register 102A electronic digital computer at the United States Naval Postgraduate School. The computer was programmed to accept a maximum sample size of 256. Outputs of the mean, standard deviation, variance, and linear correlations between the two sets of data were recorded. The correlation coefficient (r) used is defined in Hoel [1] as

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n s_x s_y}$$

where s_x and s_y are the standard deviations of the two variables involved. The first correlation attempted related the independent variable, Z_T less than 36,000 feet, to the dependent variable, $(Z_T - Z_{LMW})$, henceforth referred to as ΔZ . A sample size of 256 randomly selected points was used. This group of data was part of the 1000 points previously mentioned. A correlation coefficient of 0.66 resulted. The form of the regression equation, utilizing r is

$$\Delta Z = a Z_T + b$$

$$\text{where } a = r \frac{s_{\Delta Z}}{s_T} \text{ and } b = \Delta \bar{Z} - a \bar{Z}_T$$

The resulting equation is

$$\Delta Z = .556 Z_T - 185 \quad (\Delta Z, Z_T \text{ in } 100's \text{ ft})$$

Solving for Z_{LMW} gives

$$Z_{LMW} = .444 Z_T + 185 \quad (1)$$

The standard error is 2490 feet and can be interpreted as the Z_{LMW} distance above and below the regression line which includes about 68% of the data points. The standard error defined by

Panofsky [4] is given by the expression

$$S.E. = S_{LMW} (1 - r_{LMW}^2)^{\frac{1}{2}}$$

During the course of the graphical subtraction mentioned earlier, it was noted that there exists a marked resemblance between the contours of the LMW and the 300-mb chart. Thus, for each of the 1000 points over the two months' period, a 300-mb height was determined and plotted against Z_{LMW} on the scatter diagram shown in Figure 6. The relation here is about the same as that for Z_T versus ΔZ . However, the 300-mb versus Z_T plot does not show the radical change in slope associated with the scatter diagram of Z_T plotted against Z_{LMW} . Using only those points associated with Z_T less than 36,000 feet, the correlation coefficient, $r_{LMW,300}$, is 0.67. The regression equation, computed as before, is

$$Z_{LMW} = 2.68 Z_{300} - 464 \quad (Z_{LMW}, Z_{300} \text{ in } 100\text{'s ft}) \quad (2)$$

with a standard error of 2230 feet. The graphs of equations (1) and (2) are shown in Figure 7.

If the two independent variables Z_T and Z_{300} do not account for the same fraction of the variance of Z_{LMW} , there will be some improvement in accuracy by writing a multiple regression equation of the form

$$Z_{LMW} = a + b Z_T + c Z_{300} \quad (3)$$

where a , b , and c are constants to be determined from data in the sample. This best-fit plane may be determined by the least squares method discussed in Panofsky [4] by subtracting

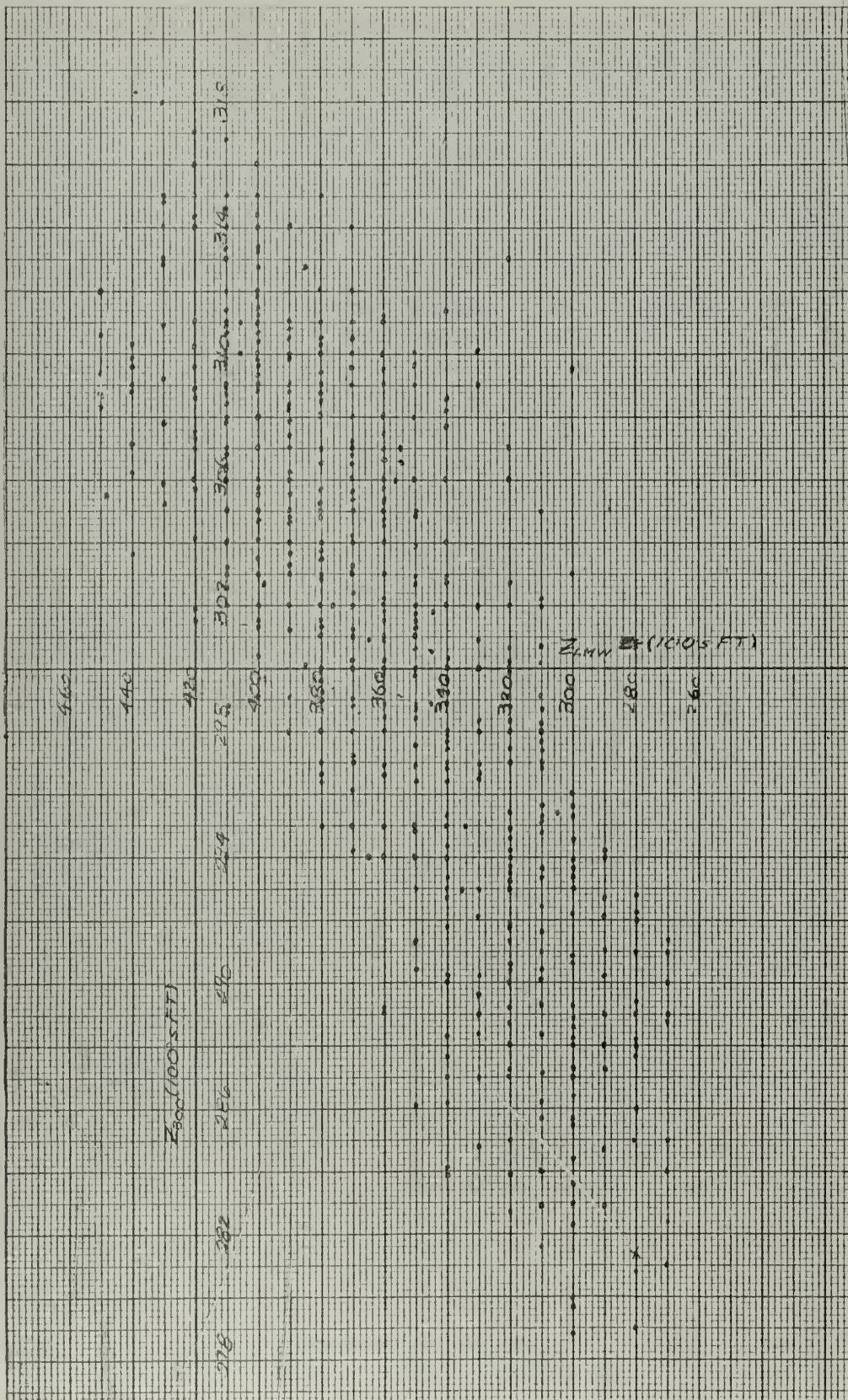


Figure 6. Scatter diagram for Z_{400} versus Z_{300}

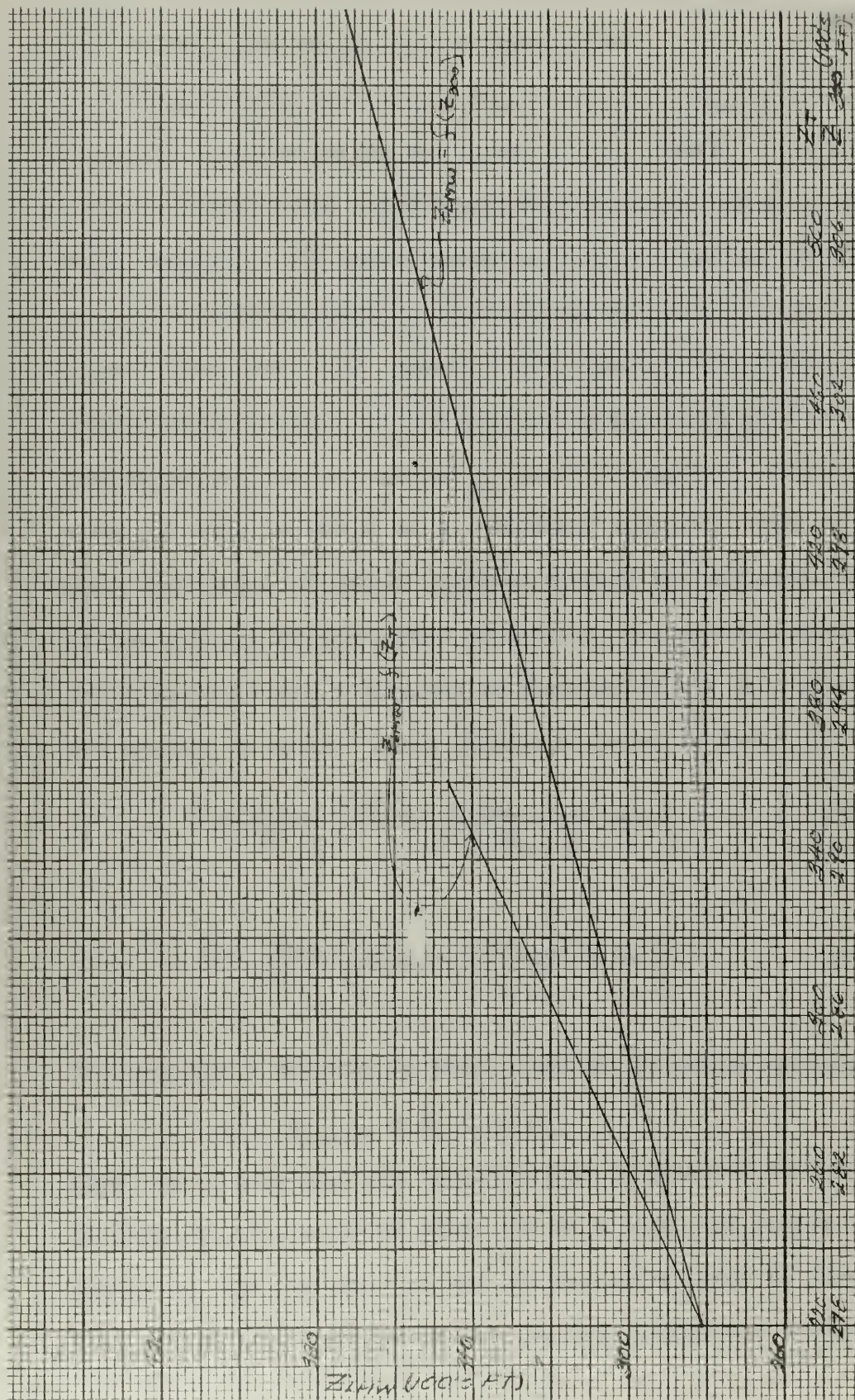


Figure 7. Graph of simple linear equations applicable to $Z_T < 36,000'$

Z_{LMW}'' , the observed value of the height of the LMW, from each side of the above equation, and squaring as

$$(Z_{LMW} - Z_{LMW}')^2 = (a + b Z_T + c Z_{300} - Z_{LMW}')^2$$

Partially differentiating the equation with respect to a, b, and c successively, gives three equations with a, b, and c as the three unknowns. All other parameters in the equation are known or can be calculated from the previously mentioned results given by the electronic computer. The simultaneous solution of these three equations in this particular case is a very arduous task requiring that an excessive number of digits be carried along in order to arrive at a reasonably accurate solution. Since the number of digits required exceeds the keyboard capacity of the ordinary Marchant desk calculator, the symbolic solution for constants b and c as given by Panofsky [4] is more practical and equally as valid. These equations are as follows:

$$b = \frac{S_{LMW}}{S_T} \left[\frac{r_{LMW,T}^2 - r_{LMW,300} r_{T,300}}{1 - r_{T,300}^2} \right] \quad (4)$$

$$c = \frac{S_{LMW}}{S_{300}} \left[\frac{r_{LMW,300}^2 - r_{LMW,T} r_{T,300}}{1 - r_{T,300}^2} \right] \quad (5)$$

The constant a may then be found from the following expression:

$$a = \bar{Z}_{LMW} - b \bar{Z}_T - c \bar{Z}_{300}$$

Thus, for a solution, the simple linear correlation coefficients between all the variables taken two at a time, the standard deviation, and the means of all the variables, must be known. This method makes maximum use of the work previously performed

by the computer and eliminates many hours of error-prone work on the desk calculator. The computer results used in calculating this multiple regression equation are shown in Tables 1 and 2.

	ΔZ	Z_T	Z_{300}
Z_T	.660	-----	-----
Z_{300}	.061	.565	-----
Z_{LMW}	-----	.580	.671

Table 1. Correlation Coefficients for $Z_T < 36,000'$

	<u>Mean</u>	<u>Standard Deviation</u>
ΔZ	-1,215	3,313
Z_{LMW}	32,338	3,009
Z_{300}	29,363	758
Z_T	31,064	3,932

Table 2. Means and Standard Deviations of all variables for $Z_T < 36,000'$ (IN FT)

Thus the regression equation (3), applicable for Z_T less than 36,000 feet, becomes

$$Z_{LMW} = 128 - .0473 Z_T + .717 Z_{300} \quad (Z_{LMW}, Z_T, Z_{300} \text{ in } 100's \text{ ft}) \quad (6)$$

Here the multiple correlation coefficient, $R_{LMW; Z_T, Z_{300}}$ is 0.713 and the standard error is 2110 feet.

Using ΔZ as the dependent variable and solving the resulting equation for Z_{LMW} gives

$$Z_{LMW} = -528 + .500 Z_T + 2.37 Z_{300} \quad (\text{ALL VARIABLES } 100's \text{ FT}) \quad (7)$$

which has a $R_{\Delta Z, Z_T, Z_{300}}$ of 0.760 but shows no improvement in the standard error because of the larger variance of the dependent variable ΔZ . Nomograms for equations (6) and (7) are shown in Figures 8 and 9.

The multiple correlation coefficient via Panofsky [4] is the square root of the following equation:

$$R = \frac{r_{LMW,T}^2 + r_{LMW,300}^2 - 2r_{LMW,T} r_{LMW,300} r_{T,300}}{1 - r_{T,300}^2}$$

The quantity R^2 gives that fraction of the total variance of the dependent variable that is accounted for by the two independent variables.

The standard error for a three-variable linear regression equation was calculated by a method given by Snedecor [6] and involves the use of the following equation:

$$S.E. = \left[\frac{(1-R^2) \sum (Z_{LMW} - \bar{Z}_{LMW})^2}{n-3} \right]^{\frac{1}{2}}$$

where n is the sample size and $n-3$ represents the number of degrees of freedom. If the sample size is large, the quantity

$$\frac{\sum (Z_{LMW} - \bar{Z}_{LMW})^2}{n-3} \text{ may be accurately represented by } \frac{\sum (Z_{LMW} - \bar{Z}_{LMW})^2}{n}$$

the variance of the dependent variable. The equation reduces simply to

$$S.E. = S_{LMW} [1-R^2]^{\frac{1}{2}}$$

The quantity $(1-R^2)$ gives that fraction of the variance that is not accounted for by the two independent variables and is commonly referred to as the coefficient of alienation.

The section of the data for which Z_T is greater than 36,000 feet has been treated in exactly the same manner as

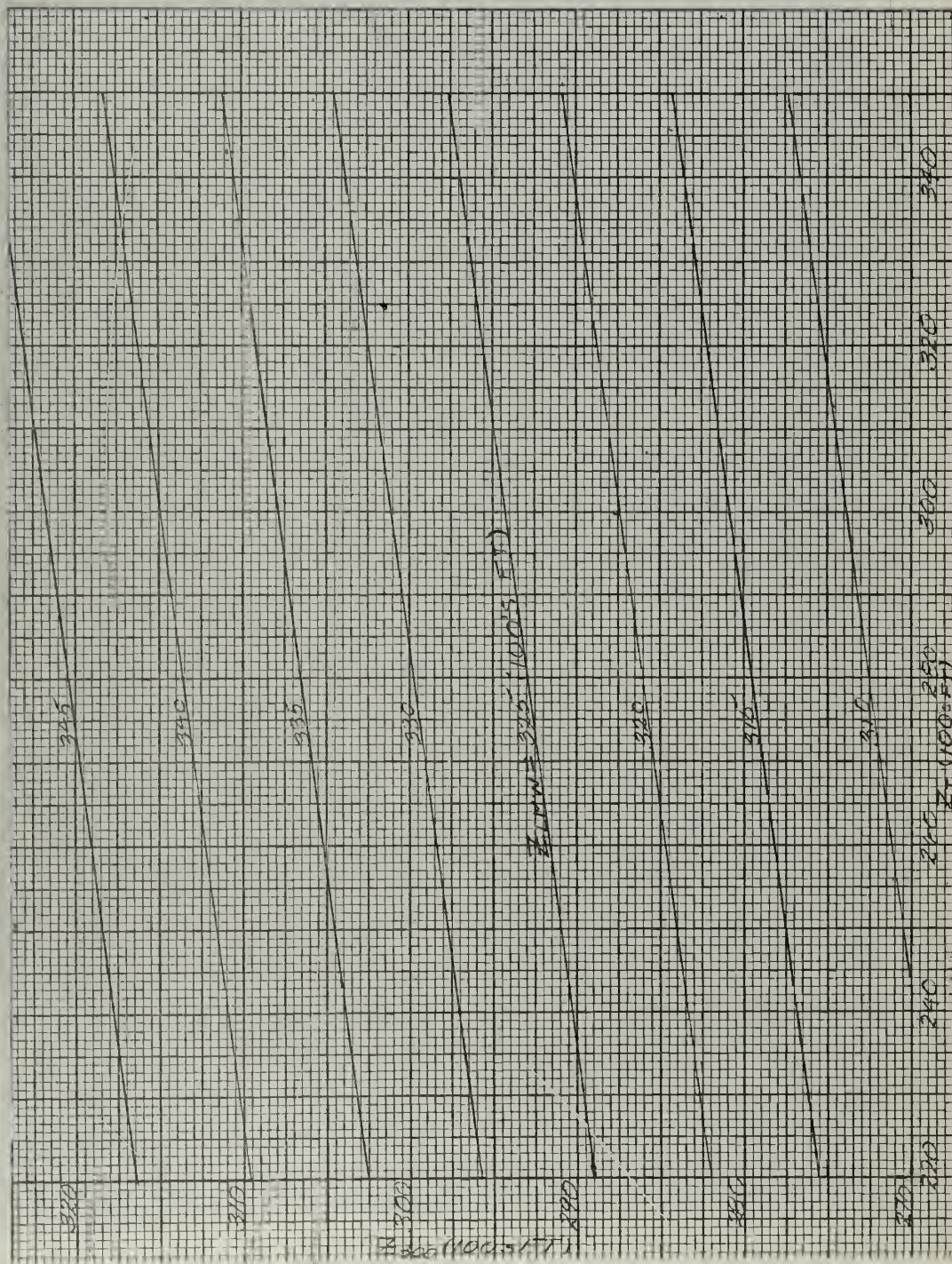
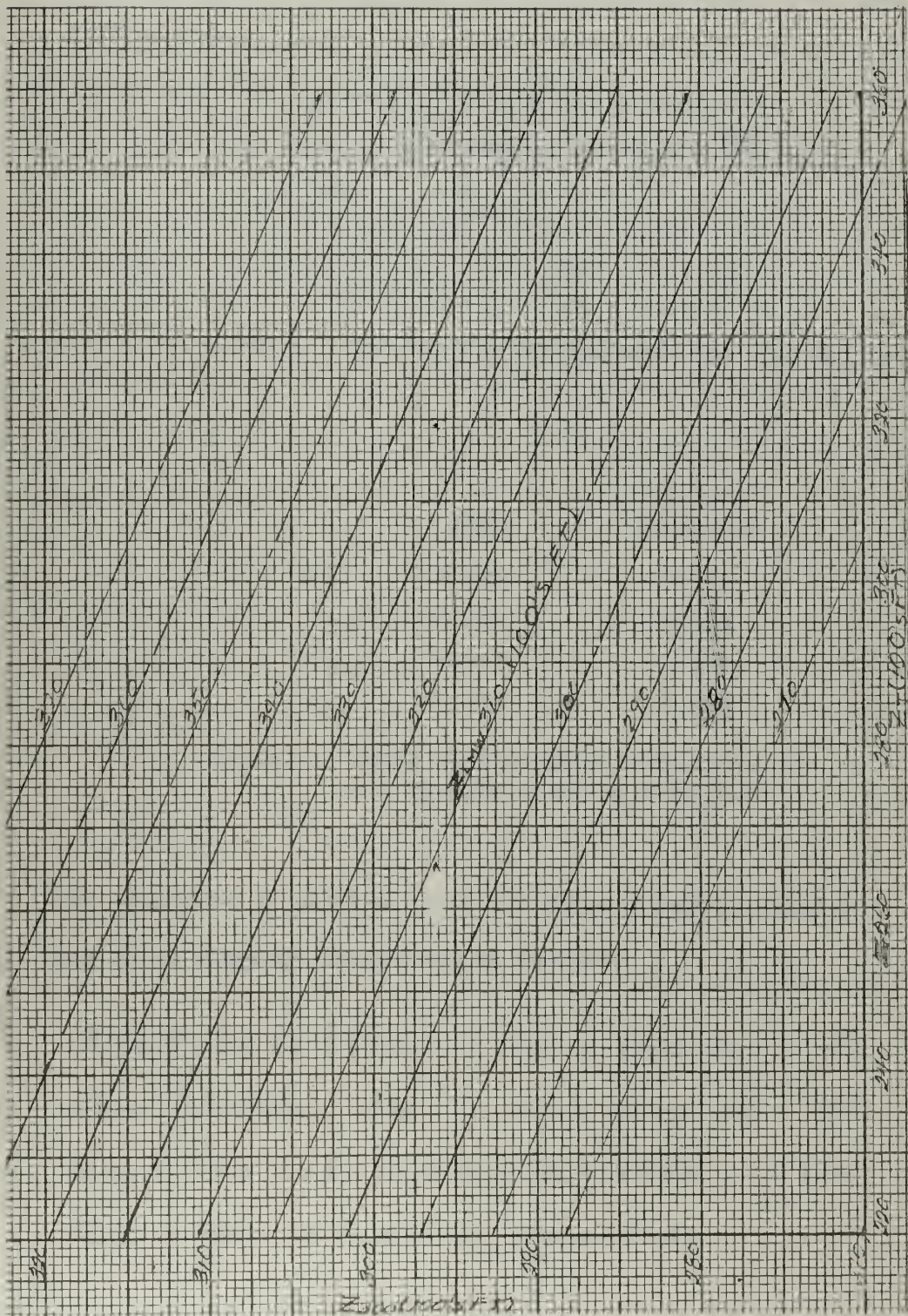


Figure 8. Nomogram for $Z_{LW} = 128 - 0.0473 Z_T + 0.717 Z_{300}$



heretofore described for Z_T less than 36,000 feet. Tables 3 and 4 summarize the electronic computer results for a sample size of 256.

	ΔZ ($Z_T - Z_{LMW}$)	Z_T	Z_{300}
Z_T	.845	-----	-----
Z_{300}	.196	.555	-----
Z_{LMW}	-----	.336	.662

Table 3. Correlation Coefficients for $Z_T > 36,000'$

	<u>Mean</u>	<u>Standard Deviation</u>
ΔZ	4,295	5,611
Z_{LMW}	38,504	3,199
Z_{300}	30,611	600
Z_T	42,771	5,865

Table 4. Means and Standard Deviations of all Variables for $Z_T > 36,000'$

The equations calculated from the data of Tables 3 and 4 and their standard errors are as follows:

$$Z_{LMW} = .194 Z_T + 302 \quad S.E. = 3010 \text{ FT} \quad (8)$$

$$Z_{LMW} = 3.53 Z_{300} - 695 \quad S.E. = 2400 \text{ FT} \quad (9)$$

$$Z_{LMW} = -123 - .2 Z_T + 1.94 Z_{300} \quad S.E. = 2395 \quad (10)$$

$$Z_{LMW} = -185 + .916 Z_T + .581 Z_{300} \quad S.E. = 2385 \quad (11)$$

Equations (8) and (11) were obtained using ΔZ as the dependent variable. The graphs of equations (8) and (9) have been drawn in Figure 10. Nomograms have been prepared for the multiple regression equations and are displayed in Figures 11 and 12.

One additional effort was made to obtain regression equations with smaller standard errors. This was attempted, using the same independent variables, by choosing data over ranges of values of the independent variables which have the most apparent linear relationship with the dependent variable. The points randomly selected for this correlation were taken from the original 1000 points and were those for which

$44,000' > Z_T > 28,000'$; $Z_{300} < 30,600'$. Figures 5 and 6 show these ranges of values to be a reasonable selection. The computer results using one sample of size 256 are summarized in Tables 5 and 6 below:

	Z_T	Z_{300}
Z_{LMW}	.694	.696
Z_{300}	.671	-----

Table 5. Correlation Coefficients for $44,000 > Z_T > 28,000'$; $Z_{300} < 30,600'$

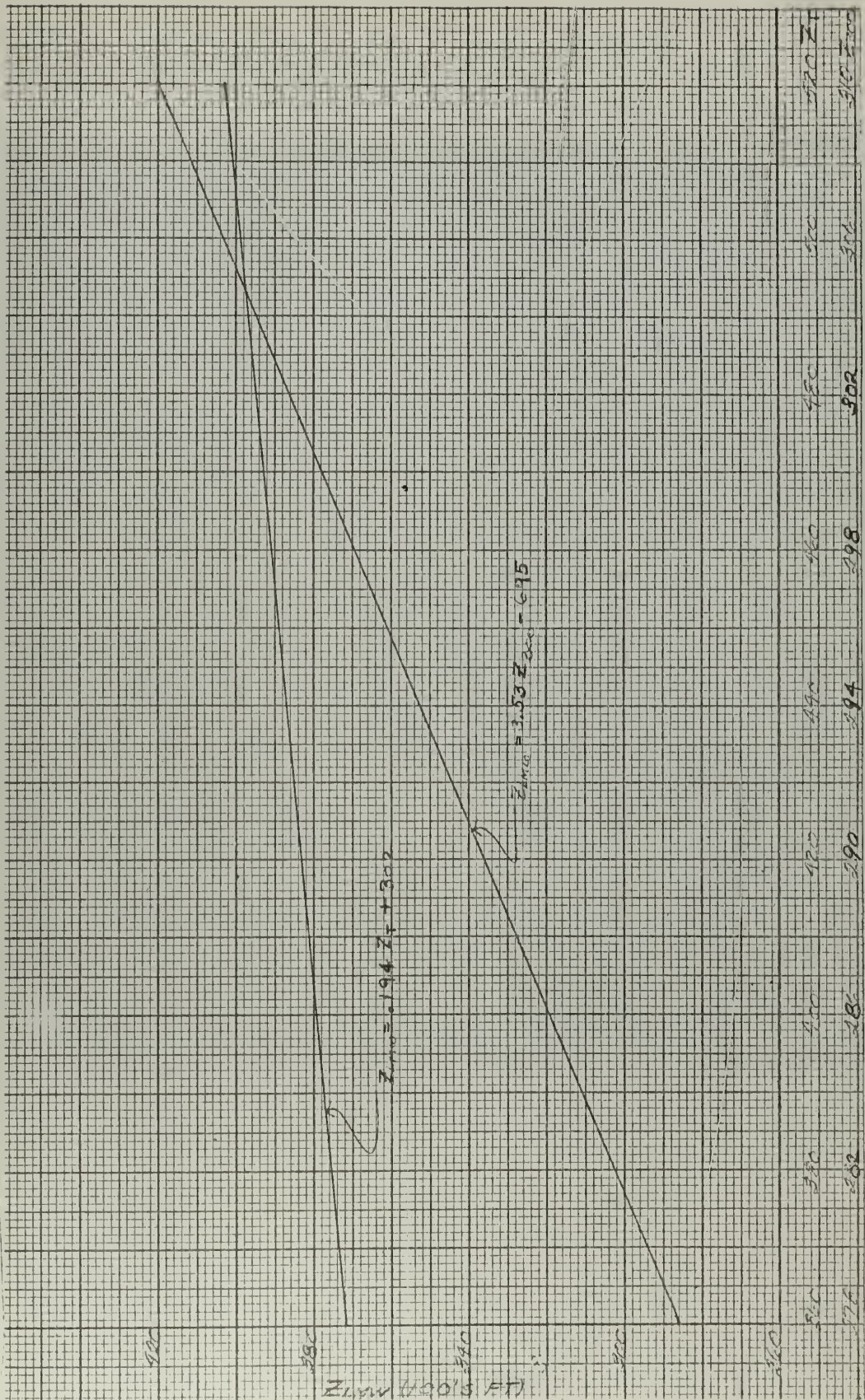


Figure 10. Graphs of simple linear equations applicable to $Z_T > 36,000'$

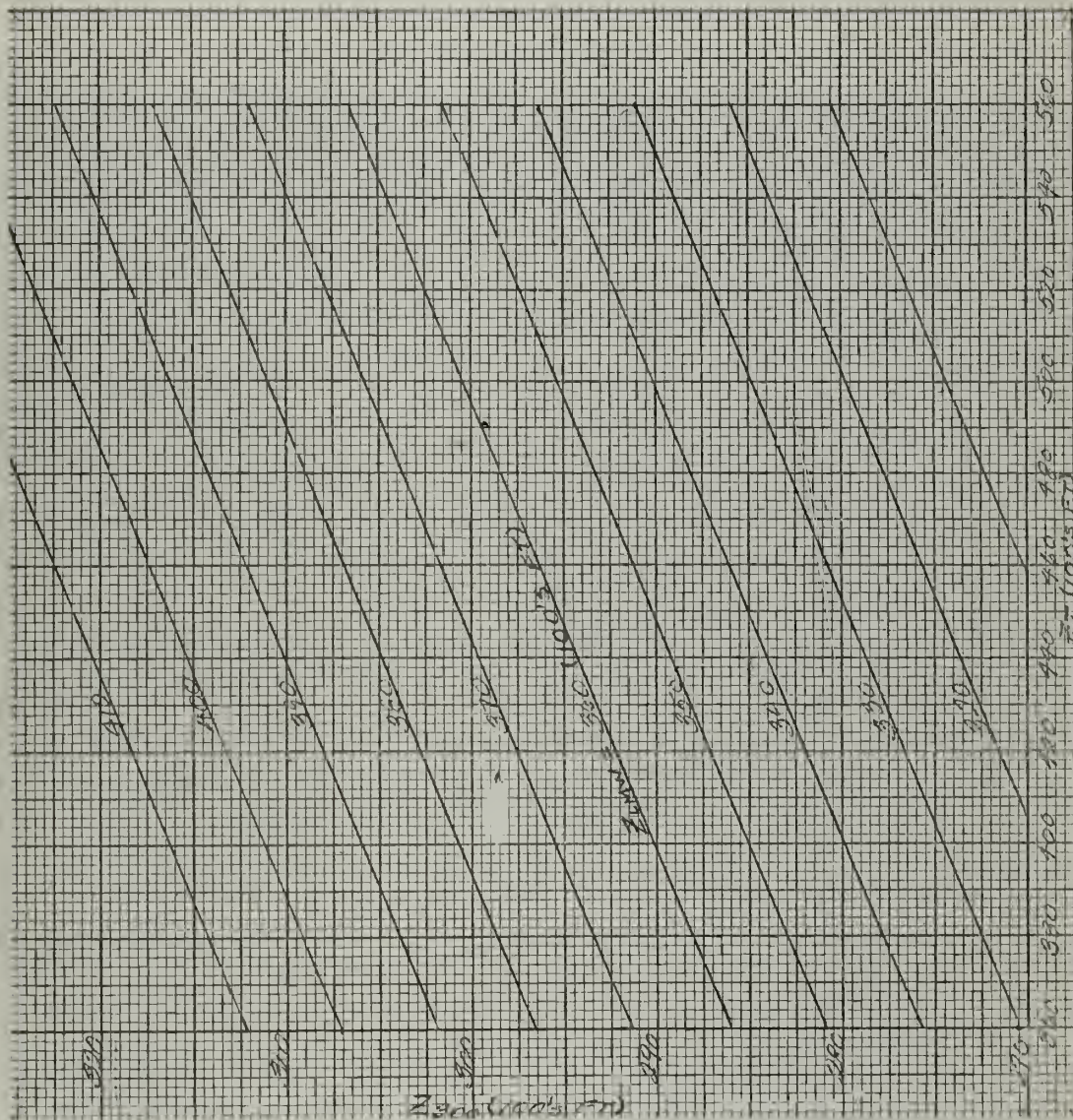


Figure 11. Nomogram for $Z_L M_0 = -123 - 1.94 Z_{300}$

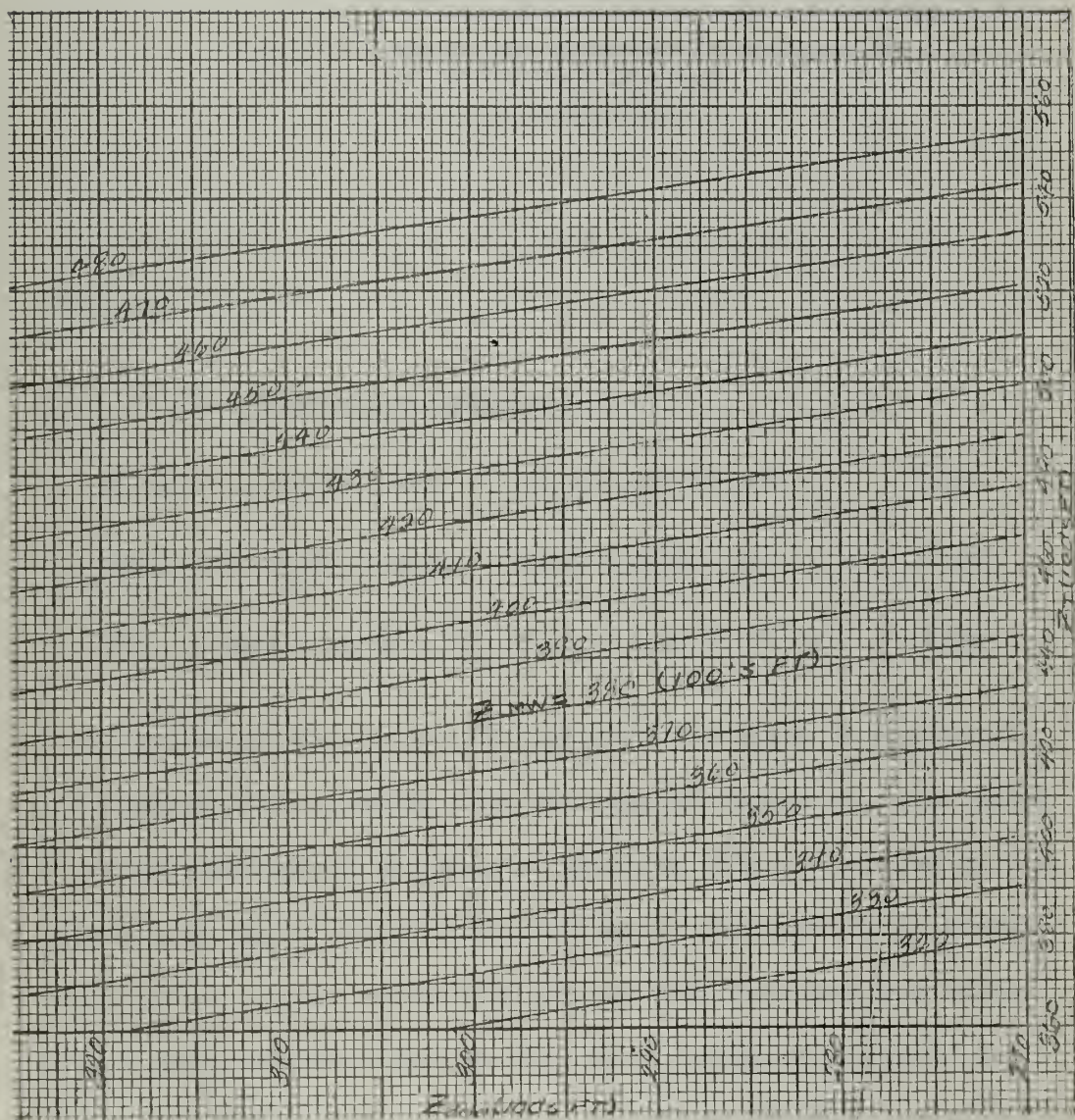


Figure 12. Nomogram for $Z_{LMW} = -185 + .916 Z_T + .581 Z_{300}$

	<u>Mean</u>	<u>Standard Deviation</u>
Z_T	36,518	3,809
Z_{LMW}	35,169	3,372
Z_{300}	29,773	594

Table 6. Means and Standard Deviations for
 $44,000 > Z_T > 28,000'$
and $Z_{300} < 30,600'$

The regression equations and their standard errors are as follows

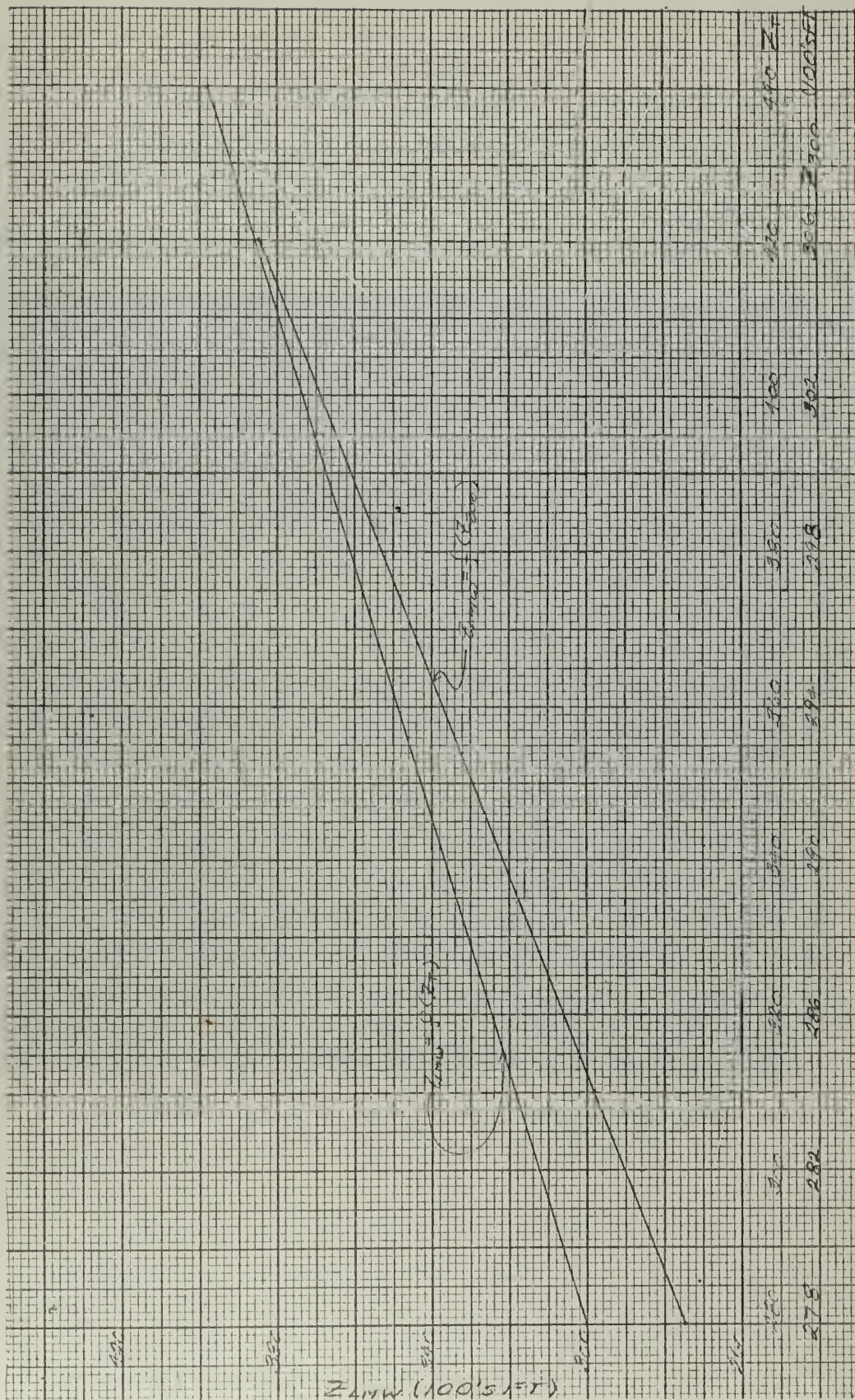
Regression Equation

$$Z_{LMW} = .614 Z_T + 128 \quad S.E. = 2430 \text{ FT} \quad (12)$$

$$Z_{LMW} = 3.93 Z_{300} - 818 \quad S.E. = 2420 \text{ FT} \quad (13)$$

$$Z_{LMW} = 283 + .021 Z_T + .207 Z_{300} \quad S.E. = 2190 \quad (14)$$

The graphs of equations (12) and (13) are drawn in Figure 13. A nomogram for equation (14) is shown in Figure 14. The value of the independent variable of equation (14) is limited by the fact that their coefficients are small, thus giving an almost horizontal plane. Equations (4) and (5) show that the values of b and c approach zero as the three linear correlation coefficients become equal. Table (5) shows the near equality of the values of r resulting from this particular selection of data.



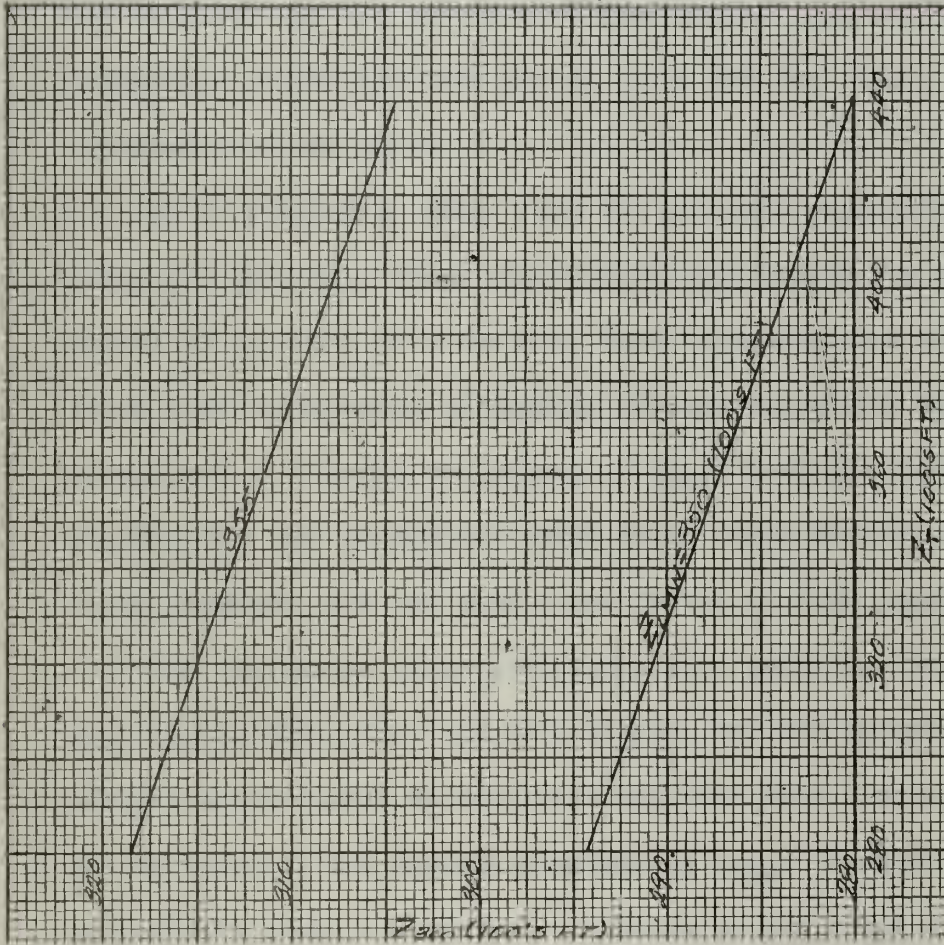


Figure 14. Nomogram for $Z_{LW} = 283 + 0.021 Z_T + 0.207 Z_{300}$

5. Conclusions and Recommendations

The assumption that $Z_T = Z_{LMW}$ is considered acceptable only where Z_T lies between 36,000 and 40,000 feet. Above and below this range, the error associated with this assumption increases rapidly. Within the continental United States, it is not uncommon for the LMW to vary from 20,000 feet below the tropopause in the southern states to 12,000 feet above the tropopause in the north.

It is concluded, then, that the multiple regression equations (7) and (11), which together are applicable for all values of Z_T , will give the best first approximation to Z_{LMW} . There will undoubtedly be weather systems which will be associated with large deviations of Z_{LMW} from the plane of regression. The conditions under which these deviations will occur, however, has not been investigated in this paper.

The regression plane associated with equation (14) is very nearly horizontal and as a result, the two independent variables have small coefficients and thus very little effect on the value of the dependent variable, Z_{LMW} . Further investigation with other independent variables in the range of applicability of this equation was prevented by insufficient time.

It was originally intended to evaluate the regression equations using independent data from the month of December 1959. However, because of the availability of an electronic computer at the United States Naval Postgraduate School, it

was felt that time would be utilized more advantageously by continuing to search for the most significant correlations among the variables under consideration. In addition, it is believed that the variety of weather systems occurring during the months of November and January, make these regression equations sufficiently general so that they may be used with confidence for independent samples of data.

Were this investigation to continue, consideration of independent variables other than Z_T and Z_{300} would certainly be warranted. Since the winds are dynamically associated with the thermal structure of the atmosphere, the temperature at the 300-mb surface may well show better association with Z_{LMW} than the height of the 300-mb surface. Some work has been done in attempting correlations with 1000-500 mb thickness values [7]. It would be logical to expect that thickness values for layers above 500 mb would give results as good or better.

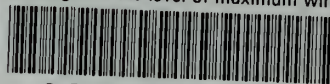
The further consideration of Z_{LMW} , Z_T and Z_{300} or any other associated variables should include serial correlations and graphical solutions of their non-linear relationships.

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The height of the level of maximum wind



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